Syllabus for Math 399, *Tutorial in Numerical Methods and Mathematical Computing*

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Fall 2024 (Revised August 26, 2024)

1 Basic Information

Class times. Monday and Wednesday, 9:30–10:45 am.

Professor's Office hours. By apppointment.

Professor's Office. Room 344, Robert S. Small Building.

Professor's Email Address. lemesurierb@charleston.edu

Professor's Websites. http://lemesurierb.people.charleston.edu/ and http://blogs.charleston.edu/lemesurierb/, but most online communication will be done through OAKS[1].

Materials. The main materials are the online Jupyter Book[2] Numerical Methods and Analysis with Python[3] and a collection of interactive Jupyter[4] notebooks.

Thus there is no text that it is necessary to buy. However, I will suggest several reference books (any of which can be borrowed from me) and online sources, and the above "book" gives references to those, for supplementary reading.

2 Course Objectives and Student Learning Outcomes

The main expectation of this course is that students learn methods for computing accurate numerical solutions to mathematical and scientific problems, and acquire an understanding of when and why particular methods work, and how reliable, accurate and efficient they are.

The first main topic is a review of Taylor polynomials, which are a basic tool in numerical computation because they allow the approximation of many functions by polynomials, which are easy to work with. Then we consider general issues of how to describe and measure the accuracy of numerical solutions, and sources of inaccuracy such as rounding in arithmetic.

[1] lms.charleston.edu/
[2] jupyterbook.org/
[3] lemesurierb.people.charleston.edu/numerical-methods-and-analysis-python
[4] jupyter.org/

Students will learn

- Methods for finding (approximately) the roots of a function.
- Methods for solving systems of simultaneous linear equations.
- Methods for approximating functions by polynomials and fitting polynomials through a collection of points.
- Methods for approximating the derivatives and definite integrals of functions.
- How to assess the accuracy of such approximations, including the effects of rounding error, and how to judge the speed of various methods.

3 Topics

In a bit more detail, the topics covered will include:

- Computing numerical approximate solutions of equations in one variable by the bisection method, fixed point iteration, Newton's method, and possibly others.
- The accuracy and limitations of standard IEEE64 computer arithmetic and the effects of rounding.
- Reliable and efficient variants of the basic row-reduction method for solving simultanous linear equations, including pivoting and the LU factorization.
- Fitting polynomials to data both by exact collocation and least squares approximation.
- Approximating functions by polynomials and piecewise polynomials, and using these to approximate derivatives and definite integrals.

As time allows we will also study one or several extra topics, such as

- Finding the minimum of a function of one variable.
- Approximating a function by trigonometric polynomials (Fourier Series).
- Computing the eigenvalues and eigenvectors of a matrix.
- Solving systems of nonlinear equations.
- Finding the minimum of a function of several variables.
- Solving ordinary differential equations.

One of these topics could be the focus of the final project.

4 Software

The main tools for computation will be Python with the packages NumPy[1] (providing numerical computing resources, in particular for linear algebra), Matplotlib[2] (for graphics), and SciPy[3] (providing further numerical computing resources, such as for solving differential equations and computing minima.) These will mostly be used within Jupyter notebooks, which provide a "literate programming" environment for combining Python code and its output with explanatory text and mathematical material.

Students might also wish to use an Integrated Development Environment [IDE] such as Spyder[4] which offers more sophisticated code editing and debugging tools.

This can all be accessed on your own computer using the free software bundle Anaconda[5] (Individual Edition). If working only with notebooks, another option is using the browser-based Colaboratory[6] system.

[1] `numpy.org`
[2] `matplotlib.org`
[3] `scipy.org`
[4] `www.spyder-ide.org/`
[5] `www.anaconda.com`
[6] `colab.research.google.com`

5 Graded Work and Grading Scheme

5.1 Assignments

There will be assignments every two weeks, combining Python coding exercises with some mathematical exercises and discussion.

5.2 Projects

There will be several projects, similar to assignments but more substantial. The topics will be decided in discussion with enrolled students; if there is more than one student, some or all of the topics may be individually chosen. (I will provide a list of suggested topics and projects.)

5.3 Grading Scheme

The total grade will be weighted 60% on the assignments, 40% on the projects.

The aggregate score guarantees at least the following grades:

A	A^-	B^+	B	B^-	C^+	C	C^-	D^+	D	D^-
90–100	87–89	84–86	80–83	77–79	74–76	70–73	67–69	64–66	60–63	57–59

6 Accommodations for Students with Disabilities

If you have a documented disability, please contact me during the first two weeks of class or as soon as you have been approved to receive accommodations, so that reasonable accommodations can be arranged. Approval for such accommodations is arranged through the Center for Disability Services: see http://disabilityservices.charleston.edu/accommodations/

7 College of Charleston Honor Code and Academic Integrity

Lying, cheating, attempted cheating, and plagiarism are violations of our Honor Code that, when identified, are investigated. Each incident will be examined to determine the degree of deception involved.

Cases of suspected academic dishonesty will be reported directly to the Dean of Students. A student found responsible by the Honor Board for academic dishonesty will receive a XXF in the course, indicating failure of the course due to academic dishonesty. This grade will appear on the student's transcript for two years after which the student may petition for the XX to be expunged. The F is permanent. The student may also be placed on disciplinary probation, suspended (temporary removal) or expelled (permanent removal) from the College by the Honor Board.

Students should be aware that unauthorized collaboration or working together without permission is a form of cheating. Unless the instructor specifies that students can work together on an assignment, quiz and/or test, no collaboration during the completion of the assignment is permitted. Other forms of cheating include possessing or using an unauthorized study aid (which could include accessing information via a cell phone or computer), copying from others' exams, fabricating data, and giving unauthorized assistance.

Students can find the complete Honor Code and all related processes in the Student Handbook at http://studentaffairs.charleston.edu/honor-system/studenthandbook/

8 Some Important Dates and Times

Monday August 26	Last day to drop/add courses.
Monday September 2	Labor Day—classes *do* meet.
Saturday and Sunday, September 21 and 22	Storm make-up days, if needed (classes will be made-up online).
Friday October 25	Last day to withdraw with a grade of "W".
Monday and Tuesday, November 4 and 5	Fall Break.
Saturday and Sunday, November 9 and 10	Storm make-up days, if needed (classes will be made-up online).
Wednesday November 27–Sunday December 1	Thansgiving Break.
Monday December 2	Last day of classes.
Tuesday December 3	Reading Day.
Wednesday December 4 to Monday December 9	Final Exam Period.